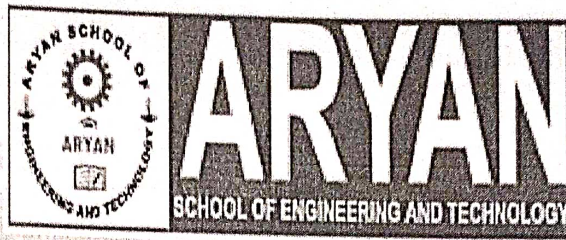


ARYAN SCHOOL OF ENGINEERING & ECHNOLOGY

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LECTURE NOTE

SUBJECT NAME- FLUID MECHANICH & HYDRAULIC MACHINE

BRANCH-MECHANICAL ENGINEERING

SEMESTER-4TH SEM

ACADEMIC SESSION-2021-22

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1.0 Properties of Fluid: —

1.1 Definition and Units of density, specific weight, specific gravity, specific volume.

1.2 Definition and Units of dynamic Viscosity, kinematic Viscosity, Surface tension, Capillary Phenomenon.

Introduction of Fluid Mechanics and Hydraulic Machine: —

Fluid Mechanics is that branch of Science which deals with the behaviour of the Fluid [liquid or gases] at rest as well as in motion.

It is up of three types: —

- i) Static Fluid
- ii) Dynamic Fluid.
- iii) kinematic Fluid.

1. Fluid static: —

The branch of Science deals with the static, kinematics and dynamic aspect of Fluid.

The study of Fluid at rest is called static Fluid.

2. kinematic Fluid: —

The study of Fluid in motion, where pressure force are not considered is called kinematic Fluid.

3. Dynamic Fluid: —

The study of Fluid is in motion, where pressure force are considered is called dynamic Fluid.

Properties of Fluid: —

1. Mass density: — Mass density of a Fluid is defined as the ratio of the mass of the Fluid to its volume.

$$\text{Mass density} = \frac{\text{Mass}}{\text{Unit Volume of Fluid}}$$

It is denoted by the symbol ρ [rho]

Unit \rightarrow kg/m^3 In SI

The density of liq may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically —

$$\text{Mass density } (\rho) = \frac{\text{Mass of Fluid}}{\text{Volume of Fluid}}$$

The value of density of water is $1 \text{ gm}/\text{cm}^3$ or $1000 \text{ kg}/\text{m}^3$

2. Specific Weight or weight density: —

Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.

\rightarrow Thus weight per unit volume of a fluid is called weight density.

\rightarrow denoted by the symbol "w".

Thus mathematically $w = \frac{\text{Weight of the Fluid}}{\text{Volume of the Fluid}}$

$$= \frac{[\text{Mass of the Fluid}] \times [\text{Acceleration due to gravity}]}{\text{Volume of the Fluid}}$$

Volume of the Fluid.

$$= \frac{\text{Mass of the Fluid} \times g}{\text{Volume of the Fluid}}$$

$$= \rho \cdot g$$

$$\boxed{w = \rho \cdot g}$$

The Value of specific weight or weight density (w) for water is 9.81×1000 Newton/ m^3 in SI Units. (10)

3. Specific Volume: —

Specific Volume of a Fluid is defined as the Volume of a Fluid occupied by a Unit mass or Volume per Unit mass of a Fluid is called Specific Volume.

Mathematically —

$$\begin{aligned} \text{Specific Volume} &= \frac{\text{Volume of Fluid}}{\text{Mass of Fluid}} \\ &= \frac{1}{\frac{\text{Mass of Fluid}}{\text{Vol}^m \text{ of Fluid}}} = \frac{1}{\rho} \end{aligned}$$

The specific Volume is the reciprocal of mass density.

Unit — m^3/kg .

Notes: — It is commonly applied to gases.

4. Specific Gravity: —

Specific gravity is defined as the ratio of the weight density of a Fluid to the weight density of a standard Fluid.

- For liquid, the standard Fluid is taken water and for
- For gases, the standard Fluid is taken as air.
- Specific gravity is also called relative density.
- dimensionless quantity.
- denoted by the symbol "S".

Mathematically: —

$$S [\text{For liquid}] = \frac{\text{Weight density of liq.}}{\text{Weight density of water.}}$$

$$S [\text{For gases}] = \frac{\text{Weight density of gas}}{\text{Weight density of air.}}$$

Thus weight density of a liq. = $S \times$ weight density of water.

$$= S \times 9.81 \times 1000 \text{ N/m}^3.$$

$$\text{density of liquid} = S \times \text{density of water}$$
$$= S \times 1000 \text{ kg/m}^3$$

If the specific gravity of liq. fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by density of water.

Notes: — Specific gravity of mercury is 13.6

$$\text{density of mercury} = 13.6 \times 1000$$
$$= 13600 \text{ kg/m}^3.$$

Eg. 1 Calculate the specific weight, density and specific gravity of 1 litre of a liquid which weight 7N.

Solⁿ → Data given as —

$$\text{Volume 1 litre} = \frac{1}{1000} \text{ m}^3.$$

$$\text{Weight 7 N} =$$

$$\text{Specific weight} = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{ m}^3}$$

$$= 7000 \text{ N/m}^3$$

(ii) Density (ρ) — $\frac{W}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = 713.5 \text{ kg/m}^3$. (111)

(iii) Specific gravity = $\frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} = 0.713$.

Eg. 2 Calculate the density, specific weight and weight of 1 litre of petrol of specific gravity 0.7.

Solⁿ → Data given as —

Volume of petrol = 1 litre.

= $1 \times 1000 \text{ cm}^3$

= $\frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Specific gravity $S = 0.7$

i) Density (ρ) = $S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = 700 \text{ kg/m}^3$.

ii) Specific weight (w) = $\rho \times g = 700 \times 9.81 = 6867 \text{ N/m}^3$.

iii) Weight (W), We know that —

Specific weight (w) = $\frac{\text{Weight}}{\text{Volume}}$.

$w = \frac{W}{V}$

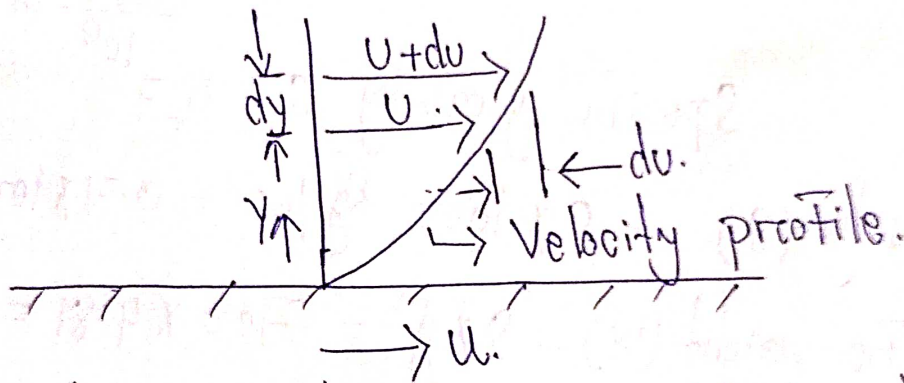
$6867 = \frac{W}{0.001}$

$W = 6867 \times 0.001 = 6.867 \text{ N}$.

Viscosity :-

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid a distance "dy" apart move one over the other at different velocities. Say U and $U + du$.

→ The viscosity together with relative velocity causes a shear stress acting betⁿ the fluid layers.



[Velocity Variation near a Solid boundary]

- The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.
- This shear stress is proportional to the rate of change of velocity with respect to y .
- It is denoted by the symbol τ (Tau).

Mathematically $\tau \propto \frac{du}{dy}$

$$\tau = \mu \cdot \frac{du}{dy}$$

Where μ = Constant of proportionality, and is known as dynamic viscosity, or only Viscosity.

From equation we have —

$$\mu = \frac{\tau}{\left(\frac{dv}{dy}\right)}$$

Thus viscosity is also defined as the shear stress require to produce unit rate of shear strain.

Units of Viscosity: —

$$\begin{aligned}\mu &= \frac{\text{Shear stress}}{\left(\frac{\text{Change of Velocity}}{\text{Change of distance}}\right)} = \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{time}}\right) \times \frac{1}{\text{Length}}} \\ &= \frac{\text{Force} \times (\text{Length})^2}{\text{time}} = \frac{\text{Force} \times \text{time}}{(\text{Length})^2}\end{aligned}$$

i) MKS Unit of Viscosity — $\frac{\text{kgf} \cdot \text{Sec}}{\text{m}^2}$

ii) CGS Unit of Viscosity — $\frac{\text{dyne} \cdot \text{Sec}}{\text{cm}^2}$

iii) SI Unit of Viscosity — $\frac{\text{Newton} \cdot \text{Sec}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$

Notes: —

The unit of Viscosity in CGS is also called poise.

$$\boxed{1 \text{ poise} = 1 \frac{\text{dyne} \cdot \text{Sec}}{\text{cm}^2}}$$

The numerical Conversion of the Unit of Viscosity From M K S Unit, to C G S Unit is given below.

$$\text{One } \frac{\text{kgf} \cdot \text{Sec}}{\text{m}^2} = \frac{9.81 \text{ N} \cdot \text{Sec}}{\text{m}^2} \quad [1 \text{ kgf} = 9.81 \text{ N}]$$

$$1 \text{ N} = 1 \text{ kg (Mass)} \times 1 (\text{m/s}^2) \text{ acceleration.}$$

$$= \frac{1000 \text{ gm} \times 100 \text{ cm}}{\text{Sec}^2} = 1000 \times 100 \frac{\text{gm} \cdot \text{cm}}{\text{Sec}^2}$$

$$= 1000 \times 100 \text{ dyne.}$$

$$[\text{dyne} = \frac{\text{gm} \cdot \text{cm}}{\text{Sec}^2}]$$

$$\frac{1 \text{ kgf} \cdot \text{Sec}}{\text{m}^2} = 9.81 \times 10,000 \frac{\text{dyne} \cdot \text{Sec}}{\text{cm}^2}$$

$$= 9.81 \times 10000 \frac{\text{dyne} \cdot \text{Sec}}{100 \times 100 \times \text{cm}^2}$$

$$= 98.1 \frac{\text{dyne} \cdot \text{Sec}}{\text{cm}^2}$$

$$= 98.1 \text{ poise.}$$

Thus for solving numerical problems, if Viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical value in M K S.

$$\text{One } \frac{\text{kgf} \cdot \text{Sec}}{\text{m}^2} = \frac{9.81 \text{ Ns}}{\text{m}^2} = 98.1 \text{ poise.}$$

$$\frac{\text{One Ns}}{\text{m}^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise.}$$

$$\boxed{1 \text{ poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}}$$

Notes: —

- 1) In SI Unit Second is represented by "s" and Not "Sec".
- 2) If Viscosity is given in poise, it must be divided by 10 to get its equivalent numerical value in SI units.

Sometimes a unit of Viscosity as centipoise is used where

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise.}$$

$$\boxed{1 \text{ c.p.} = \frac{1}{100} \text{ P}}$$

The viscosity of water at 20°C is 0.01 poise or 1.0 Centipoise.

Kinematic Viscosity: —

It is defined as the ratio betⁿ the dynamic Viscosity and density of Fluid.

→ It is denoted by the greek symbol "ν" called "nu".

$$\text{Mathematically - } \nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

Thus Units of kinematic viscosity is obtained as: —

$$\begin{aligned} \nu &= \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} \\ &= \frac{\text{Force} \times \text{time}}{\text{Mass/Length.}} = \frac{\text{Mass} \times \frac{\text{Length}}{(\text{time})^2} \times \text{time}}{[\text{Mass/Length}]} \\ &= (\text{Length})^2 / \text{Time.} \end{aligned}$$

In MKS and SI, the unit of kinematic Viscosity is $\frac{(\text{Meter})^2}{\text{s}}$.

C.G.S Unit $\rightarrow \text{cm}^2/\text{Sec}$.

Notes: —

* In C.G.S units, kinematic Viscosity is also known as stoke.

$$\text{Thus One stoke} = \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4} \text{m}^2/\text{sec}.$$

$$\text{Centistoke means} = \frac{1}{100} \text{Strooke}.$$

Surface Tension and Capillarity: —

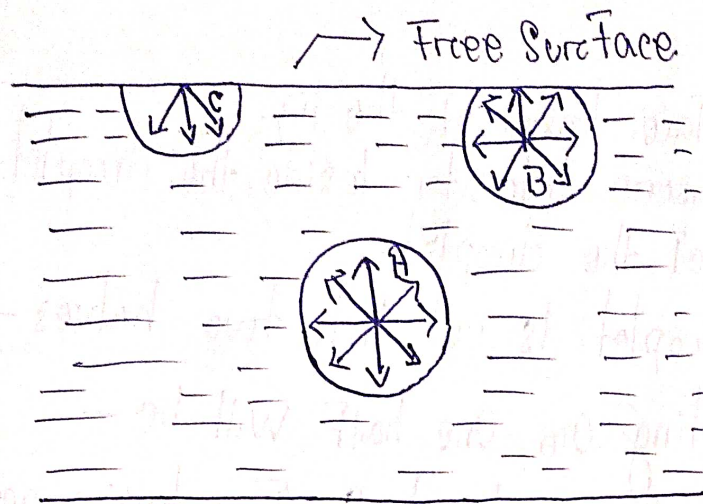
Surface tension is defined as the tensile force acting on the surface of the liquid in contact with a gas or on the surface betⁿ two immiscible liquid such that the contact surface behaves like a membrane under tension.

\rightarrow The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area.

\rightarrow It is denoted by the greek letter " σ ".

Units — In MKS $\rightarrow \text{kgf/mtr}$.

In SI $\rightarrow \text{N/mtr}$.



- Consider three molecules A, B, C of a liq. In a mass of liq.
- The molecule A is attracted in all directions equally by the surrounding molecules of the liq.
- Thus the resultant force acting on the molecule A is zero.
- Molecule "B" which is situated near the Free Surface is acted upon by upward and downward forces which are unbalanced.
- A net resultant force on molecule "B" is acting in the downward direction.
- The molecule "C" situated on the Free Surface of liq. does experience a resultant downward force.
- All the molecules on the Free Surface experienced a downward force.
- Thus the Free Surface of the liq. acts like a very thin film under tension of the surface of the liq. act as through it is an elastic membrane under tension.

Surface tension on liq. Droplet:

Consider a small spherical droplet of a liq. of radius " r " on the entire surface of the droplet. the tensile forces due to surface tension will be acting?

Let σ = Surface tension of the liq.

P = pressure Intensity Inside the droplet.

d = dia of the droplet.

Let the droplet is cut into two halves —

The forces acting on one half will be —

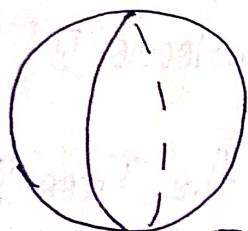
i) Tensile forces due to Surface tension, acting around the circumference of the cut portion.

$$= \sigma \times \text{Circumference}$$

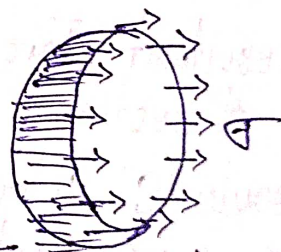
$$= \sigma \times \pi d.$$

(ii) pressure force on the area —

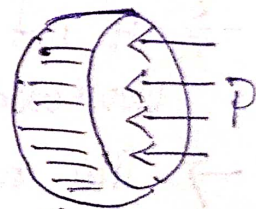
$$\frac{\pi}{4} d^2 = P \times \frac{\pi}{4} d^2$$



[Droplet]



[Surface tension]



[pressure force]

These two forces will be equal & opposite under equilibrium condition.

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d.$$

$$P = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}.$$

Surface tension in a hollow bubble: —

A hollow bubble like a soap bubble in air has two surfaces in contact with air. One inside and other outside.

Thus two surfaces are subjected to surface tension.

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

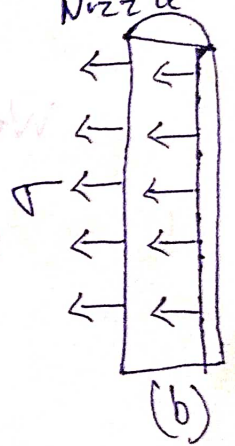
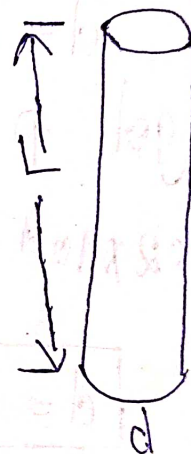
$$p = \frac{2 \times \sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$

Surface tension on a liq. jet: —

[Stream of fluid that is projected into a surrounding medium]
Nozzle

Consider a liq. jet of diameter "d" and length "L".

Let $p = p_{int}$ intensity inside the liq. jet above the outside pressure.



σ = Surface tension of the liq.

Consider the equilibrium of the semi jet, we have —

$$\begin{aligned} \text{Force due to } p_{int} &= p \times \text{area of semi jet.} \\ &= p \times L \times d. \end{aligned}$$

$$\text{Force due to surface tension} = \sigma \times 2L$$

Equating the force,

$$p \times L \times d = \sigma \times 2L$$

$$p = \frac{\sigma \times 2L}{L \times d}$$

Eg. 1 The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure. Calculate the diameter of the droplet of water.

Solⁿ → Data given as —

Surface tension (σ) = 0.0725 N/m.

Pressure intensity P in excess of outside pressure is —

$$P = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

d = dia of the droplet.

We get $P = \frac{4\sigma}{d}$

$$0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$$

$$d = 1.45 \text{ mm.}$$

Eg. 2 Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.

Solⁿ → Data given as —

dia of bubble $d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$

Pressure in excess of outside $P = 2.5 \text{ N/m}^2$

For a soap bubble, using eqn. we get —

$$P = \frac{8\sigma}{d}$$

$$2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}} \Rightarrow \sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m.}$$

$$= 0.0125 \text{ N/m.}$$

Eg. 3 The prc outside the droplet of water of dia 0.04 mm is 10.32 N/cm^2 . Calculate the prc within the droplet if surface tension is given as 0.0725 N/m of water.

Solⁿ → Data given as —

dia of the droplet — $d = 0.04 \text{ mm} = 0.04 \times 10^{-3} \text{ m.}$

pressure outside the droplet = 10.32 N/cm^2 .

$$= 10.32 \times 10^4 \text{ N/m}^2.$$

Surface tension. $\sigma = 0.0725 \text{ N/m.}$

The prc inside the droplet, in excess of outside prc is given by eqn.

$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \text{ N/m}^2.$$

$$= \frac{7250}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2.$$

pressure inside the droplet —

= $p + \text{prc. outside the droplet.}$

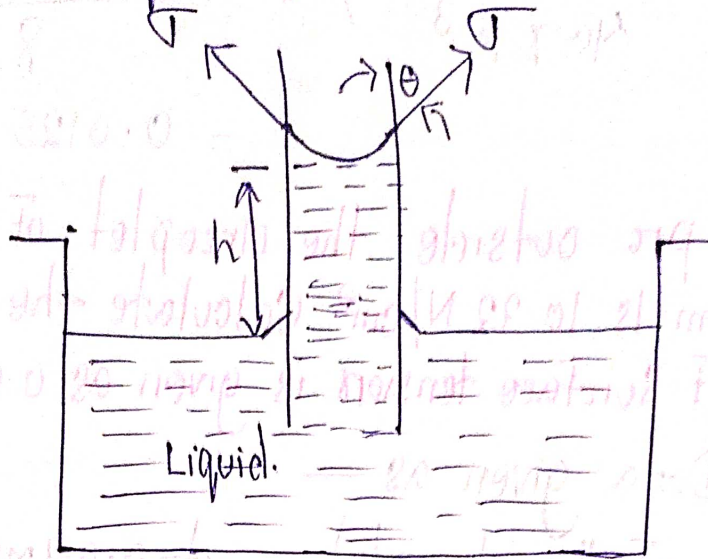
$$= 0.725 + 10.32$$

$$= 11.045 \text{ N/cm}^2.$$

Capillarity : —

→ Capillarity is defined as a phenomenon of rise or fall of a liq surface in a small tube relative to the adjacent general level of liq, when the tube is held

Vertically In liq.



The rise of liq surface is known as capillary rise while the fall of the liq surface is known as capillary depression.

→ Unit - cm or mm of liq.

→ Its value depends upon the specific weight of the liq.

ii) diameter of the tube.

iii) Surface tension of the liq.

Assignment

i) what is mass and weight density.

ii) what do you mean by FMHM and its classification?

iii) what is specific volume.

iv) what is specific gravity.

v) what is viscosity.

vi) what is kinematic viscosity.

vii) what is surface tension.

viii) what is capillarity.

Chapter - II Fluid Pressure & Its measurement

Syllabus: —

2.1 Definition and Units of Fluid pressure, pressure intensity and pressure head.

Fluid pressure at a point: —

Consider a small area dA in large mass of fluid.

→ If the fluid is stationary then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA .

→ Let " dF " is the force acting on the area dA in the normal direction.

→ Then the ratio of dF/dA is known as the intensity of pressure.

→ It is represented by " p ".

$$p = dF/dA$$

If the force is uniformly distributed over the area.

(A) then pressure at any point is given by —

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$$

Force or pressure Force = $F = p \times A$

Units of pressure are -

1) kgf/m^2 and kgf/cm^2 — M.K.S.

2) N/m^2 or Newton/ m^2 — S.I

N/m^2 is known as pascal represented by "Pa"

$\text{kPa} = \text{kilo pascal} = 1000 \text{ N}/\text{m}^2$.

$1 \text{ bar} = 100 \text{ kPa} = 10^5 \text{ N}/\text{m}^2$.

Q.1 A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.

Solution \rightarrow Data given as —

dia of ram = $D = 30 \text{ cm} = 0.3 \text{ m}$.

" " plunger $d = 4.5 \text{ cm} = 0.045 \text{ m}$.

Force on plunger $F = 500 \text{ N}$.

Weight = W

Area of ram = $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.0706 \text{ m}^2$.

Area of plunger $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = 0.00159 \text{ m}^2$.

Pressure intensity due to plunger = $\frac{\text{Force of plunger}}{\text{Area of plunger}}$.

$$= \frac{500}{0.00159} \text{ N}/\text{m}^2$$

due to Pascal's law the intensity of pressure will be equally transmitted in all directions.

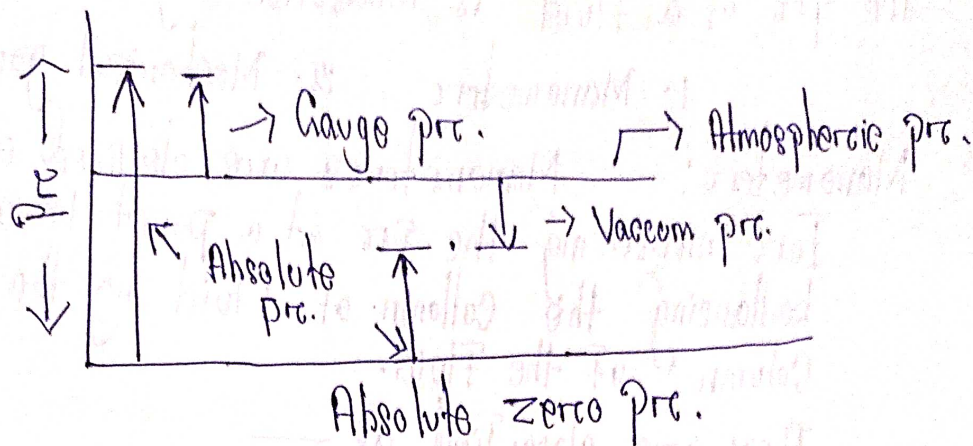
$$\text{Pre Intensity of ram} = \frac{500}{.00159} = 314465.4 \text{ N/m}^2.$$

$$\text{Pre Intensity at ram} = \frac{Wt}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{.07068 \text{ m}^2}$$

$$\frac{W}{0.07068} = 314465.4.$$

$$\boxed{W = 22.222 \text{ kN.}}$$

Absolute, Gauge, atmospheric and Vacuum prc: —



i) Atmospheric Prc: —

The atmospheric air exerts a normal prc upon all surfaces with which it is in contact and known as atm prc.

ii) Absolute Prc: —

It is defined as the prc which is measured with reference to absolute vacuum prc. or absolute zero prc.

iii) Gauge Prc: —

It is defined as the prc which is measured with the help of a prc measuring instrument in which the atm prc is less taken as datum. The atm prc in the scale is marked as zero.

iv) Vacuum Prc: —

It is defined as the prc below the atm prc.

Mathematically: —

$$\text{abs prc} = \text{atm prc} + \text{gauge prc.}$$

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$\text{Vacuum Prc} = \text{atm prc} - \text{abs prc.}$$

$$P_{\text{vacuum}} = P_{\text{atm}} - P_{\text{abs}}$$

Pressure measuring instrument:

The P_{rc} of a fluid is measured by following devices - Pie-

1. Manometer
2. Mechanical gauge.

1) Manometer: — Manometers are defined as the device for measuring the P_{rc} at a point in a fluid by balancing the column of fluid by the same another column of the fluid.

They are classified as —

1) Simple Manometer. 2) Differential Manometer

2) Mechanical gauge: — Mechanical gauges are defined as the device used for measuring the P_{rc} by balancing the fluid column by the spring or dead weight, cement.

Used mechanical P_{rc} gauges are —

i) Diaphragm P_{rc} gauge.

ii) Bourdon tube P_{rc} gauge.

iii) Dead weight P_{rc} gauge.

iv) Bellows P_{rc} gauge.

Simple Manometer: — A simple manometer of a glass tube has one of its ends connected to a point where P_{rc} is to be measured and other end remains open to atm.

Common type of simple manometers are —

i) piezometer.

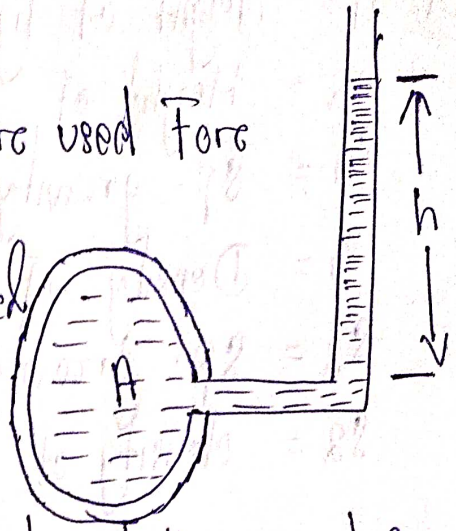
ii) U-tube Manometer.

iii) Single column Manometer.

Piezometer: —

→ It is the simple form of Manometer used for measuring gauge pr.

→ One end of the Manometer is connected to the point where pr is to be measured and other end is open to the atm.



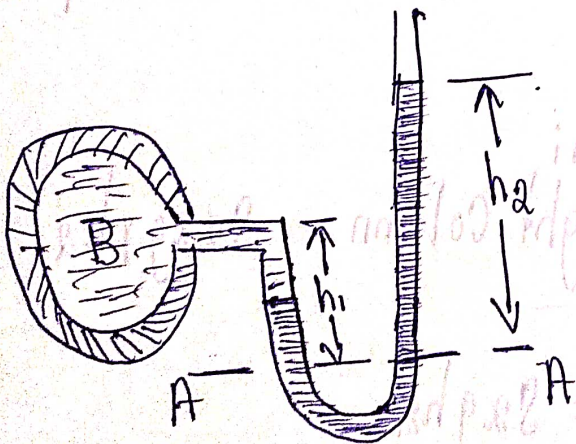
→ The rise of liq gives the pr head at the point A.

→ Then pr at A $P_A = \rho \cdot g \cdot h$

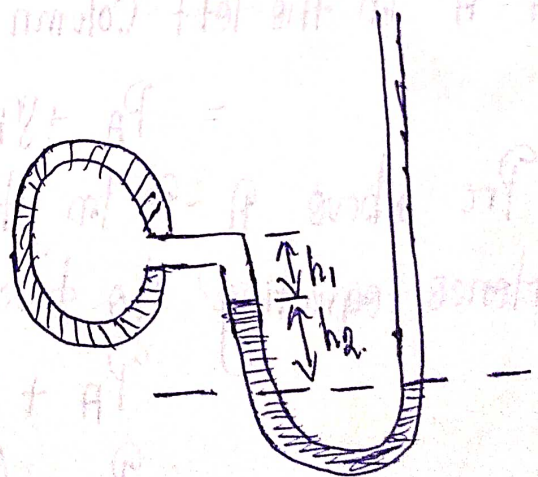
U-tube Manometer: —

It consists of glass tube bent in U-shape. One end of which is connected to a point at which pr is to be measured and other end remains open to the atm.

→ The tube generally contains mercury.



(a) For gauge pr.



(b) For Vacuum Pr.

(a) For gauge pr: —

Let be the point which is to be measured, whose value is P . The datum line is A-A.

Let h_1 = Height of light liq above the datum line.

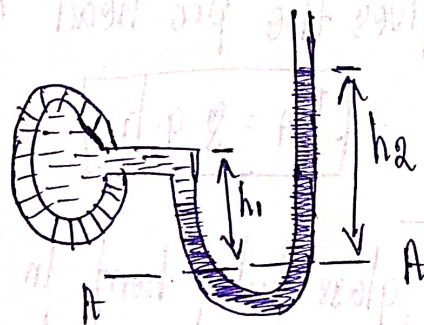
h_2 = Height of heavy liq above the datum line.

S_1 = Sp. gravity of liq light.

S_1 = Density of light liq = $1000 \times S_1$

S_2 = Sp. gravity of heavy water.

S_2 = density of heavy weight = $1000 \times S_2$.



(a) Force gauge prc.

Prs is same in a horizontal surface. Hence prs above the horizontal datum surface line A-A in the left column and in the right column of U-tube manometer should be same prs above A-A in the left column.

$$= P_A + S_1 \times g \times h_1$$

Prs above A-A in the right column = $S_2 \times g \times h_2$.

Hence equating the two prs —

$$P_A + S_1 g h_1 = S_2 g h_2$$

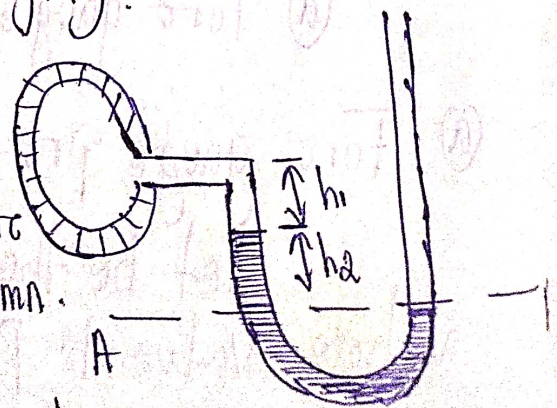
$$P_A = (S_2 g h_2 - S_1 g h_1)$$

(b) Force Vacuum prc: —

Force measuring vacuum prc the level of the heavy liq in the manometer. Then prs above A-A in the left column.

$$S_2 g h_2 + S_1 g h_1 + P_A$$

Prs head in the right column above A-A



Prs above A-A in the left column —

$$\rho_2 g h_2 + \rho_1 g h_1 + P$$

Prs head in the right column above A-A = 0

$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$$

$$P = -[\rho_2 g h_2 + \rho_1 g h_1]$$

Q. The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp gr 1.80 g. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the prs of fluid in the pipe if difference of mercury level in the two limbs is 20 cm.

Solⁿ → Data given as —

SP gravity of fluid $S_1 = 0.9$

Density of fluid = $\rho_1 = S_1 \times 1000$

$$= 0.9 \times 1000$$

$$= 900 \text{ kg/m}^3$$

SP gr of mercury = 13.6

Density of mercury = $\rho_2 = 13.6 \times 1000 = 13600 \text{ kg/m}^3$

Difference of mercury level $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

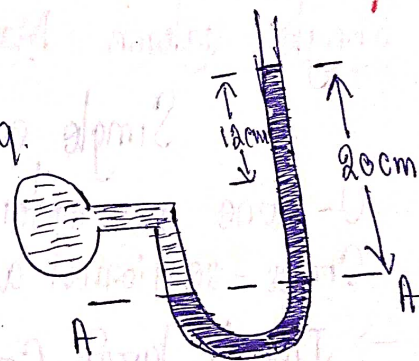
Height of fluid from A-A $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let P = Prs of fluid in pipe.

Equating the above prs. we get —

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P + 900 \times 9.81 \times 0.08 = 13600 \times 9.81 \times 0.2$$



$$\begin{aligned}
 P &= (13.6 \times 1000 \times 9.81 \times 0.2) - (900 \times 9.81 \times 0.08) \\
 &= 25977 \text{ N/m}^2. \\
 &= 2.597 \text{ N/cm}^2. \quad \underline{\text{Ans}}
 \end{aligned}$$

Q.2

Assignment:

A Small U-tube manometer containing mercury is connected to a pipe in which fluid of sp. gr. 0.8, and having vacuum pres is flowing. The other end of the manometer is open to atm. Find the vacuum pres in pipe, if the difference of mercury level in the two limb is 40cm and the height of fluid in the left from centre of pipe is 15cm below.

Single column Manometer: —

Single column Manometer is a modified form of U-tube manometer. In which a reservoir having a large cross-sectional area.

→ Due to large cross sectional area of the reservoir, for any variation of pres the change in the liq level in the reservoir will be very small which may be neglected and hence the pres is given by the height of liq in the other limb.

→ The other limb may be vertical or inclined.

→ Thus there are two types of Single column Manometer—

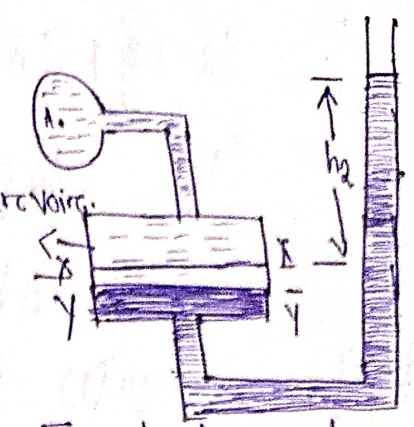
i) Vertical Single column Manometer.

ii) Inclined Single column Manometer.

Vertical Single Column Manometers: —

Vertical Single Column Manometer

Let $x-x$ be the datum line in the reservoir and $y-y$ the right limb of the manometer. When it is not connected to the pipe.



→ When the manometer is connected to the pipe, due to high P_{rc} at A, the heavy liq. in the reservoir will be pushed downward and will rise in right limb.

[Vertical Single Column Manometer]

- $A h_1$ = Fall of heavy liq. in reservoir.
- $a h_2$ = Rise of heavy liq. in right limb.
- h_1 = Height of centre of pipe above $x-x$.
- P_A = P_{rc} at A, which is to be measured.
- A = Cross-sectional area of the reservoir.
- a = Cross-sectional area of the light limb.
- S_1 = Sp. gravity of liq. in pipe.
- S_2 = Sp. gravity of heavy liq. in reservoir & right limb.
- ρ_1 = Density of liq. in pipe.
- ρ_2 = density of liq. in reservoir.

Fall of heavy liq. in reservoir will cause a rise of heavy liq. level in the right limb.

$$A \times h_1 = a \times h_2$$

$$\boxed{h_1 = \frac{a \times h_2}{A}} \quad \text{--- (1)}$$

Now consider the datum line $y-y$ as shown in fig.

Then P_{rc} in the right limb above $y-y = \rho_2 \times g \times (A h_2 + h_1)$

P_{rc} above the left limb above $y-y = \rho_1 \times g \times (A h_1 + h_1)$

Equating these prc we have: —

$$\rho_2 \times g \times (A h_2 + h_2) = \rho_1 \times g \times (A h_1 + h_1) + P_A$$

$$P_A = \rho_2 g (A h_2 + h_2) - \rho_1 g (A h_1 + h_1)$$

$$= A h [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g.$$

but from eqn (1)

$$A h = \frac{\alpha x h_2}{A}$$

$$P_A = \frac{\alpha x h_2}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g.$$

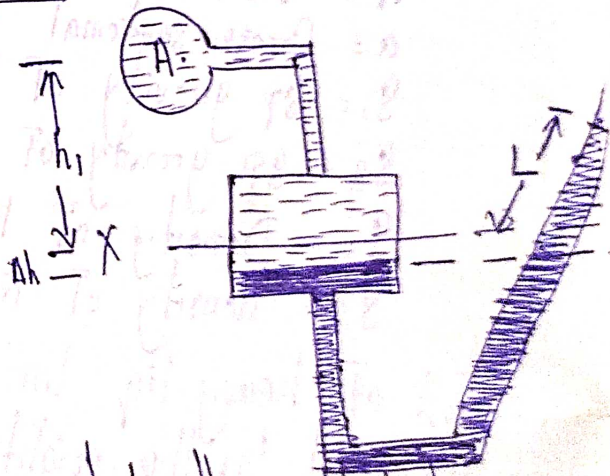
As the area A is very large as compared to a, hence ratio α/A becomes very small and can be neglected.

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

Inclined Single Column Manometers:

Inclined Single Column Manometers is more sensitive.

→ Due to inclination the distance moved by the heavy liq in the right limb will be more.



Let L = Length of heavy liq moved in the right limb.

θ = Inclination of right limb with horizontal.

h_2 = Vertical rise of heavy liq in right limb from $x-x$ = $L \sin \theta$.

From the eqn the prc at A —

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g.$$

Substituting the value of h_2 we get.

Q. A Single Column Manometer is Connected to a pipe Containing a liq. of sp. gr. 0.9. Find the prc in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading. The sp gr of mercury is 13.6.

Solution \rightarrow Data given as —

$$S_1 = 0.9.$$

$$S_1 = 900 \text{ kg/m}^3.$$

$$S_2 = 13.6$$

$$S_2 = 13.6 \times 1000.$$

Density \rightarrow

$$\frac{\text{Area of reservoir}}{\text{Area of the right limb}} = \frac{A}{a} = 100.$$

Height of the liq. $h_1 = 20 \text{ cm} = 0.2 \text{ m}.$

Rise of mercury in the right limb = $h_2 = 40 \text{ cm} = 0.4 \text{ m}.$

$$P_A = P_C \text{ in pipe.}$$

Using eqn we get —

$$P_A = \frac{a}{A} h_2 [S_2 g - S_1 g] + h_2 S_2 g - h_1 S_1 g$$

$$= \frac{1}{100} \times 0.4 [13600 \times 9.81 - 900 \times 9.81] + 0.4 \times 13600 \times 9.81 - 0.2 \times 900 \times 9.81$$

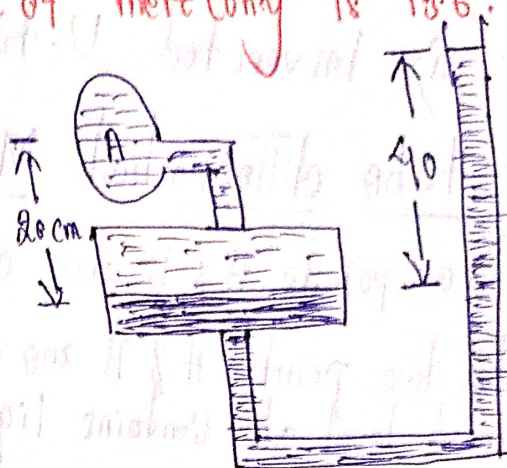
$$= 52134 \text{ N/m}^2$$

$$= 5.21 \text{ N/cm}^2.$$

Differential Manometer:

Differential Manometers are the device used for measuring the difference of prc betⁿ two points in a pipe or in two different pipes.

\rightarrow A differential manometer consist of a U-tube containing a heavy liq, whose two ends are connected to the points.



whose difference of pres is to be measured.

Most Commonly types of differential manometers: —

i) U-tube differential manometer.

ii) Inverted U-tube differential Manometer.

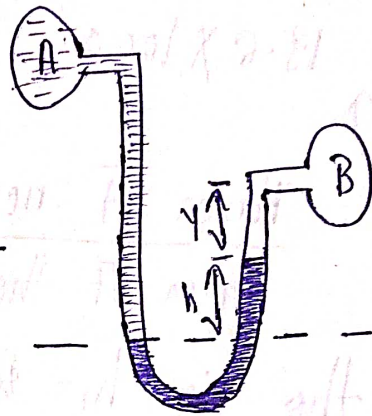
i) U-tube differential Manometer: —

Two points A & B are at different level —

Let the two points A & B are at different level also contains liq of different sp. gr.

→ These points are connected to the U-tube differential manometer.

→ Let the pres at point A & B are P_A & P_B .



Let h = difference of mercury level in the U-tube.

y = distance of the center of B from the mercury level in the right limb.

S_1 = density of liq at A.

S_2 = " " " at B.

S_g = " " heavy liq or mercury.

Taking datum line at x-x.

Pres above x-x in the limb = $S_1 g (h+x) + P_A$

where pres P_A = Pres at A.

Pres above x-x in the right limb = $S_g g x h + S_2 g x y + P_B$

where pres P_B = Pres at B.

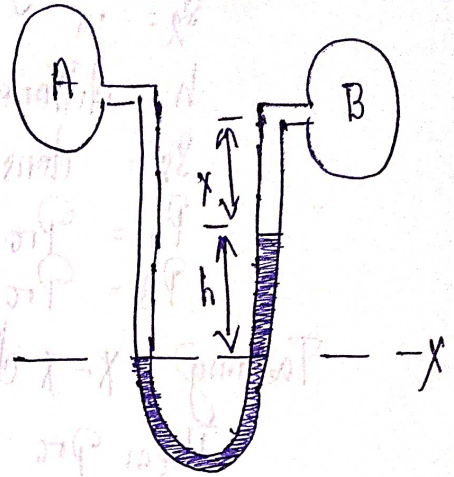
Equating the two pres, we have —

$$S_1 g (h+x) + P_A = S_g g x h + S_2 g x y + P_B$$
$$P_A - P_B = S_g g x h + S_2 g x y + P_B$$

$$= h \times g [S_2 - S_1] + S_2 g y - S_1 g x.$$

difference of P_{rc} at A & B -

Two points A & B are at same level
 In the given Fig A & B are the same level and contains the same liq. of density S_1 .



P_{rc} above x-x in right limb -

$$S_2 \times g \times h + S_1 \times g \times x + P_B$$

P_{rc} above x-x in left limb -

$$P_A \times g \times (h+x) + P_A.$$

Equating the two P_{rc} : -

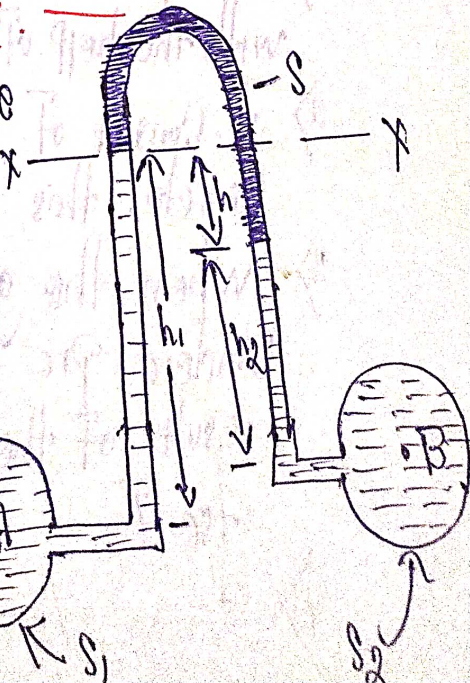
$$S_2 \times g \times h + S_1 \times g \times x + P_B = P_A \times g \times (h+x) + P_A$$

$$P_A - P_B = S_2 \times g \times h + P_A \times g \times (h+x) - P_A \times g \times (h+x)$$

$$= g \times h (P_B - P_A).$$

Inverted U-tube differential Manometers:

It consists of an inverted U-tube containing a light liq. The two ends of the U-tube are connected to the points whose difference of P_{rc} is to be measured.



→ It is used for measuring difference of low P_{rc} .

→ Inverted U-tube manometer connected to the point A & B.

→ Inverted U-tube manometers connected to the points A & B
 Let the P_{rc} at A is more than the P_{rc} at B.

Let h_1 = Height of liq. in the left limb below datum line $x-x$.

h_2 = Height of liq. in the right limb.

ρ_1 = density of liq. at A.

ρ_2 = " " " " B.

h = difference of light liq.

ρ_s = density of light liq.

P_A = P_{rc} at A.

P_B = P_{rc} at B.

Taking $x-x$ datum line: —

Then P_{rc} in the left limb below $x-x$ = $P_A - \rho_1 g h_1$

" " " right " " $x-x$ = $P_B - \rho_2 g h_2 - \rho_s g h$

Equating the two P_{rc} —

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_s g h$$

$$P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h$$

Burden tube P_{rc} gauge: —

- 1) The P_{rc} above or below the atm P_{rc} may be easily measured with the help of burden tube P_{rc} gauge.
- 2) It consists of an elliptical tube ABC bent into an arc of a circle. This bent up tube is called burden tube P_{rc} .
- 3) When the gauge tube is connected to the C, the fluid under P_{rc} flows into the tube of the burden tube as a result of the increased P_{rc} tube tends to straighten itself.
- 4) Since the tube is increased in a circular

$$\frac{P_A - P_B}{\rho_0 g} = x \left[\frac{\rho_0}{\rho} - 1 \right]$$

$$h = x \left[\frac{\rho_h}{\rho_0} - 1 \right]$$

Case - II. If the differential manometer contains a liq. lighter than the liq. flowing through the pipe.

where $\rho_1 = \rho$ sp. gravity of lighter liq. in U-tube manometer.

$\rho_0 =$ " " " Fluid flowing through in U-tube "

$x =$ difference of lighter liq. columns in U-tube.

The value of h is given by —

$$h = x \left[1 - \frac{\rho_1}{\rho_0} \right]$$

Case - III → Inclined Venturimeter with differential U-tube manometer. Let the differential manometer contains heavier liq.

Then h is given as —

$$h = \left[\frac{P_1}{\rho g} + z_1 \right] - \left[\frac{P_2}{\rho g} + z_2 \right] = x \left[\frac{\rho_h}{\rho_0} - 1 \right]$$

Case - IV — Similarly for Inclined Venturimeter in which differential manometer contains a liq. which is lighter than the liq. flowing through the pipe. Then —

$$h = \left[\frac{P_1}{\rho g} + z_1 \right] - \left[\frac{P_2}{\rho g} + z_2 \right]$$

$$h = x \left[1 - \frac{\rho_1}{\rho_0} \right]$$

Limitations: —

1) Bernoulli's eqn has been derived under the assumption that no external force except the gravity force is acting on the liq. But in actual practice some external forces always act on the liq. when they affect the flow of liq.

2) If the liq. is flowing in a curved path the energy due to centrifugal force should also be taken into account.

Pitot-tube: — It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

→ It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to conversion of the kinetic energy into pressure energy.

The pitot tube consists of a glass tube bent at right angles. Considering two points 1 & 2 at the same level, such as stage 2 is the inlet of pitot-tube and 1 is the far away from tube.

Let $P_1 =$ Pressure at point 1.

$V_1 =$ Vel of fluid at point 1.

$P_2 =$ Pressure at point 2.

$V_2 =$ Vel of fluid at point 2.

$H =$ Depth of tube in the liq.

$h =$ Rise of the liq in the tube above free surface.

Applying Bernoulli's theorem —

$$P_1/\rho g + v_1^2/2g + z_1 = P_2/\rho g + \frac{v_2^2}{2g} + z_2 \quad \left| \begin{array}{l} \frac{P_1}{\rho g} = H \\ \frac{P_2}{\rho g} = h \end{array} \right.$$

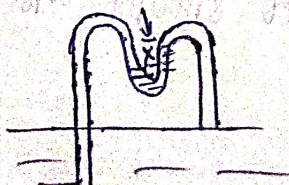
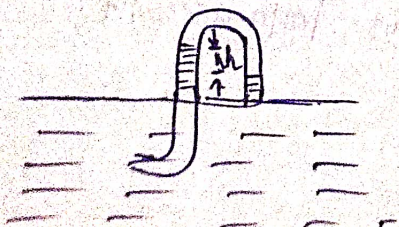
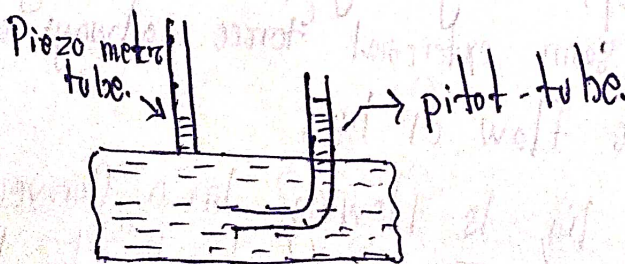
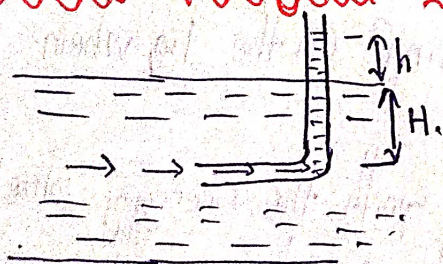
$$H + \frac{v_1^2}{2g} + z_1 = h + H$$

$$v_1 = \sqrt{2gh}$$

Actual Velocity

$V_{act} = C_v \sqrt{2gh}$, where $C_v =$ Co-efficient of pitot.

Different arrangement of pitot-tube: —



7
 Eg. 1 Water is flowing through a pipe of 50 cm dia under prc of 29.43 N/cm² and with mean velocity 2 m/s. Find the total head or total prc energy per unit weight of the water at a cross-section which is 5 m above the datum line.

Solⁿ → Data given as:—

$$\text{dia of pipe } (d) = 50 \text{ cm} = 0.5 \text{ m.}$$

$$\text{Pressure } P = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/cm}^2$$

$$\text{Velocity } (V) = 2 \text{ m/s.}$$

$$\text{Datum head } z = 5 \text{ m.}$$

$$\text{Total head} = \text{Prc head} + \text{kinetic head} + \text{datum head.}$$

$$= \frac{P}{\rho g} + \frac{V^2}{2g} + z_1$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} + 5$$

$$= 35.204 \text{ m.}$$

Eg. 2 The water is flowing through a pipe having diameter 20 cm & 10 cm at sec 1 & 2 respectively. The rate of flow through pipe is 35 lit/s. The section 1 is 6 m above datum and sec 2 is 4 m above datum. If the prc at sec 1 is 39.24 N/cm². Find the intensity of prc at section 2.

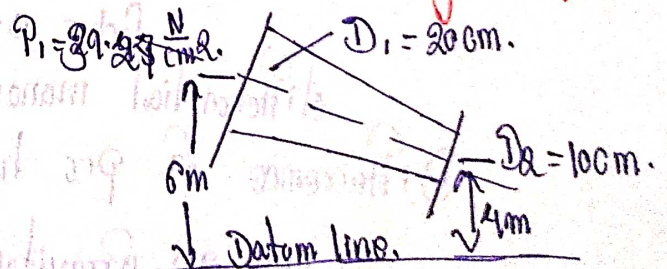
Solⁿ → At section 1 →

$$D_1 = 20 \text{ cm} = 0.2 \text{ m.}$$

$$A_1 = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2.$$

$$P_1 = 39.24 \text{ N/cm}^2.$$

$$z_1 = 6 \text{ m.}$$



At section - 2. $D_2 = 0.1 \text{ m,}$

$$A_2 = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2$$

$$z_2 = 4 \text{ m.}$$

$$P_2 = ?$$

$$\text{Rate of Flow } Q = 35 \text{ lit/s.} = 35 = 0.035 \text{ m}^3/\text{s}$$

Now, $Q = A_1 V_1 = A_2 V_2$.

$$V_1 = Q/A_1 = 0.35/0.0314 = 1.114 \text{ m/s.}$$

$$V_2 = Q/A_2 = 0.035/0.00785 = 4.456 \text{ m/s.}$$

Applying Bernoulli's eqn at section 1 & 2, we get —

$$P_1/\rho g + v_1^2/2g + z_1 = P_2/\rho g + v_2^2/2g + z_2$$

$$\frac{39.24 \times 10^4}{10000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 0 = \frac{P_2}{10000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.$$

$$P_2 = 40.27 \text{ N/cm}^2$$

Eg. 3. A horizontal Venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet & throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.

Soln → Data given as: —

dia of inlet - $d_1 = 30 \text{ cm} = 0.3 \text{ m}$.

$$a_1 = \frac{\pi}{4} (0.3)^2 = 706.85 \text{ cm}^2.$$

dia of throat - $d_2 = 15 \text{ cm}$

$$a_2 = \frac{\pi}{4} (15)^2 = 176.7 \text{ cm}^2.$$

$$C_d = 0.98.$$

differential manometer = $x = 20 \text{ cm}$ of mercury.

Difference of p/c head $h = x \left[\frac{S_h}{S_o} - 1 \right]$

$S_h = \text{sp gravity of mercury} = 13.6$

$S_o = \text{sp " " water} = 1$

$$h = 20 \left[\frac{13.6}{1} - 1 \right] = 20 \times 12.6 = 252 \text{ cm of water.}$$

discharge through Venturimeter is given by eqn —

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \frac{706.85 \times 176.7}{\sqrt{706.85^2 - 176.7^2}} \times \sqrt{2 \times 9.81 \times 252} = 125$$

Orifices: — Orifices is a small opening of any cross-section [such as triangular, rectangular etc] on the side or at the bottom of a tank, through which a fluid is flowing.

→ it is used for measuring the rate of flow of fluid.

Applying Bernoulli's theorem at 1 & 2 —

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2, \quad H + 0 = 0 + \frac{v_2^2}{2g}, \quad v_2 = \sqrt{2gh}$$

Orifices Coefficient: — The coefficients of Orifices are —

1) Co. of Velocity C_v , 2) Co. of Contraction C_c , 3) Co. of discharge C_d .

1) Co. of Velocity C_v — it is defined as the ratio betⁿ the actual vel of a jet of liq at Vena Contracta and the theoretical vel of jet.

→ denoted by C_v .

$$C_v = \frac{\text{Actual vel of jet at Vena-Contracta}}{\text{Theoretical Velocity}} = \frac{V}{\sqrt{2gh}}$$

where $V =$ actual vel.

$C_v =$ Co. of vel.

$\sqrt{2gh} =$ theoretical vel.

C_v ranges 0.95 to 0.99. For diff orifices depends on shape & size.

ii) Co. of Contraction (C_c) — it is defined as the ratio of the area of the jet at Vena-Contracta to the area of the orifice.

→ denoted by C_c .

$$C_c = \frac{a_c}{a}$$

$a =$ area of the orifice.

$a_c =$ area of the Vena-Contracta. $C_c = \frac{\text{area of the jet at Vena-Contracta}}{\text{area of the orifice}}$

The value of C_c varies from 0.61 to 0.69 depending on shape & size of orifice.

Co. of discharge: — it is the ratio of actual discharge from an Orifice to the theoretical discharge from the orifice.

→ denoted by C_d .

→ If Q is actual discharge & Q_{th} is theoretical discharge then.

$$C_{cl} = \frac{Q_{act}}{Q_{th}}$$

$$C_d = \frac{act\ vel \times act\ area}{Th.\ vel \times Th.\ area}$$

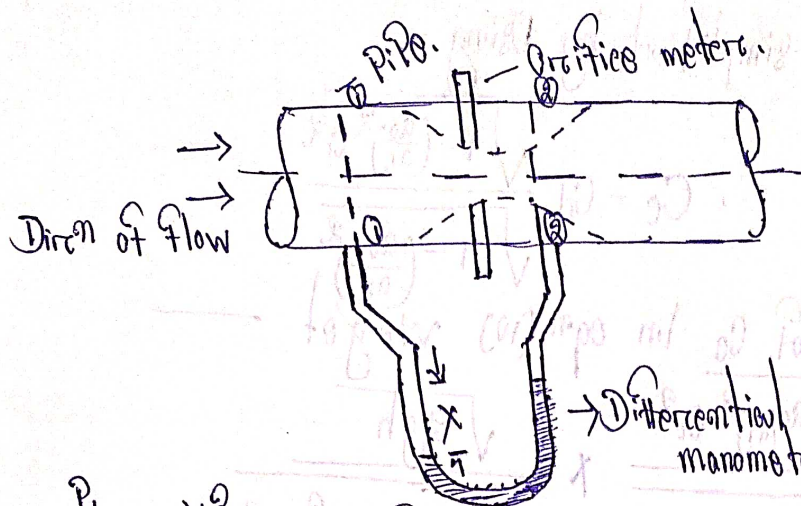
$$C_{cl} = C_v \cdot C_c$$

Range: 0.61 to 0.65 but for general purpose 0.62.
Classification: —

- i) According to the size: —
 - 1) Small orifice [if the head of liq above the centre of orifice is more than 5 times the depth of orifice]
 - 2) Large orifice [if head is less than 5 times the depth of orifice]
- ii) According to shape: —
 - 1. Circular. 2. Triangular 3. Rectangular 4. Square.
- iii) According to the shape of upstream edge: —
 - 1. Sharp edged orifice 2. Bell mounted orifice.
- iv) According to nature of discharge: —
 - 1. Free discharge orifice.
 - 2. Downed or submerged orifices —
 - i) partially submerged orifice ii) Fully submerged orifice.

Orifices meters or Orifices plate: —

- i) It is a device used for measuring the rate of flow of fluid through a pipe.
- ii) It is cheaper as compared to Venturimeter.
- iii) It also works on same principle with Venturimeter.
- iv) It consists of a flat circular plate which has a sharp edge called orifice, which is concentric with the pipe.
- v) The orifice dia is kept generally 0.5 times the dia of pipe, though it may vary 0.4 to 0.8 times the pipe dia.



Let $P_1 = P_2$ at Section (1)

$V_1 = V_2$ " " "

$a_1 =$ Area of pipe at Section (1).

a_2, V_2, P_2 are corresponding values at Sec (2).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2.$$

$$\left[\frac{P_1}{\rho g} + z_1 \right] - \left[\frac{P_2}{\rho g} + z_2 \right] = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}.$$

but $\left[\frac{P_1}{\rho g} + z_1 \right] - \left[\frac{P_2}{\rho g} + z_2 \right] = h =$ differential head.

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{or} \quad 2gh = V_2^2 - V_1^2 \quad V_2 = \sqrt{2gh + V_1^2} \quad \text{--- (i)}$$

Now Section (2) is at the Vena Contracta and a_2 represents the area at the Vena Contracta.

If a_0 is the area of orifice then we have $C_c = \frac{a_2}{a_0}$.

Where $C_c =$ Co. of Contraction. $a_2 = a_0 \times C_c$ --- (ii)

By Continuity eqn — $a_1 V_1 = a_2 V_2$ or $V_1 = \frac{a_2}{a_1} V_2 = \frac{a_0 C_c}{a_1} V_2$ --- (iii)

Substituting the values of V_1 in eqn (i) we get —

$$V_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 V_2^2}{a_1^2}}$$

$$V_2^2 = 2gh + \left(\frac{a_0}{a_1} \right)^2 C_c^2 V_2^2$$

$$V_2^2 \left[1 - \left(\frac{a_0}{a_1} \right)^2 C_c^2 \right] = 2gh$$

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 C_c^2}}$$

$$\text{Discharge } Q = V_2 \times a_2 = V_2 \times a_0 C_c = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 C_c^2}}$$

The above expression is simplified by using —

$$C_d = C_c \sqrt{1 - \left(\frac{a_0}{a_1}\right)^2} \quad , \quad C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\left(\frac{a_0}{a_1}\right)^2}$$

Substituting this value of C_c in eqn (w) we get —

$$R = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\left(\frac{a_0}{a_1}\right)^2} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

Notes → Where, $C_d = C_o$ of discharge for Orifice meter.

The value of C_d is less as compared to Venturimeter.

Flow through pipe chapters - 5

(13)

Pipe: — Pipe is a closed conduit. Generally of circular cross-section used to carry water or any other fluid.

- when the pipe is running full, the flow is under pr but if the pipe is not running full the flow is under pr [culverts sewer pipes]

Loss of Fluid Friction: — The frictional resistance of a pipe depends upon the roughness of the inside surface of the pipe more the roughness more the resistance.

→ Friction is known as fluid friction and the resistance is known as frictional resistance.

According Friction:

- i) Frictional resistance varies with sq of the velocity.
- ii) Frictional resistance varies with natural surface.
- iii) Among various laws, the Darcy-Weisbach formula & Chezy's formula.

Loss of energy in pipes:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy is lost —

Energy losses:

1) Major Energy losses — Due to friction it is calculated by a Darcy-Weisbach formula pipe & Chezy's formula.

2) Minor Energy losses — Due to sudden expansion of pipe.

u) Sudden contraction of pipe.

u) Bend in pipe.

u) Pipe fittings etc.

u) An obstruction of pipe.

Darcy - Weisbach Formula: — The loss of head in pipes due to friction

Calculated by Darcy Weisbach eqn.

$$h_f = \frac{4flv^2}{2gd}$$

where h_f = loss of head due to friction.

f = Co. of friction [Function of Reynolds numbers]

$$= 16/Re \text{ For } Re < 2000 \text{ [Viscous Flow]}$$

$$= \frac{0.079}{Re^{1/4}} \text{ For } Re \text{ Varying From } 4000 \text{ to } 10^6.$$

L = Length of the pipe.
 V = Mean vel of flow.
 D = dia of the pipe.

Chozy's Formula:

$$h_f = \frac{F_l}{8g} \times \frac{P}{A} \times L \times V^2$$

h_f = loss of head due to Friction.

P = wetted perimeter of pipe.

A = C.S area of pipe.

L = Length of pipe.

V = Mean vel of Flow.

$$M = \frac{A}{P} = \frac{\text{area of flow}}{\text{perimeter}} = \text{Hydraulic mean depth or hydraulic radius.}$$

$$M = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = d/4.$$

Substituting $P/A = 1/M$,

$$h_f = \frac{F_l}{8g} \times \frac{1}{M} \times L \times V^2$$

$$V^2 = h_f \times \frac{8g}{F_l} \times M \times \frac{1}{L}$$

$$V = \sqrt{\frac{F_g}{F_l} \times M \times \frac{h_f}{L}}$$

$\sqrt{F_g/F_l} = C$, where C is constant known as Chozy's constant.

$h_f/L = i$ loss of head per unit length.

Substituting $V = C \sqrt{M i}$ value of M is always $d/4$.

Hydraulic Gradient line:

i) It is defined as the line which gives the sum of per head $\frac{P}{w}$ static head (z).

ii) If a flowing fluid in a pipe w.r.t the reference line or it the line which is obtained by joining of the top of the vertical coordinate showing per head (P/w) of a flowing fluid in a pipe from the centre of the pipe.

iii) It is briefly written as H.G.L.

Total Energy line:

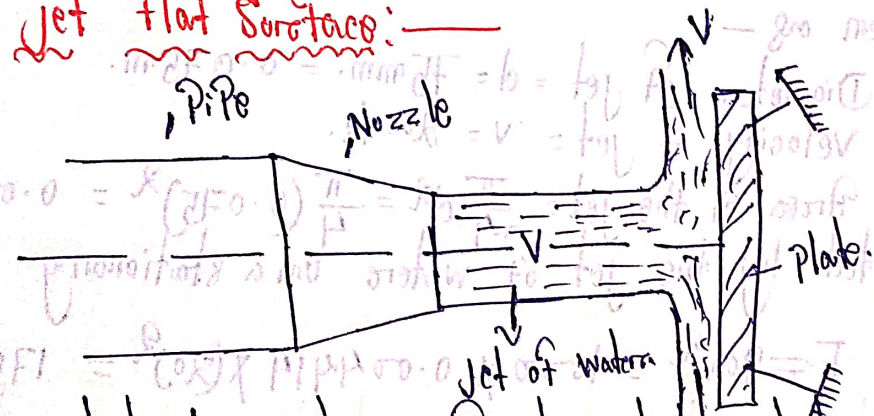
i) It is defined as the line which gives the sum of per head, static head & kinetic head, of a flowing fluid w.r.t to some reference line or it is the line which obtained by joining the tops of all vertical coordinates showing sum of per head, static head & kinetic head.

Introduction — Impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

Various Cases of Impact of Jet are —

1. Force exerted by the jet on a stationary plate when —
 - i) plate is vertical to the jet.
 - ii) plate is inclined to the jet.
 - iii) plate is curved.
2. Force exerted by the jet on a moving plate when —
 - i) plate is vertical to the jet.
 - ii) plate is inclined to the jet.
 - iii) plate is curved.

Impact of Jet Flat Surface:



Force exerted by jet on fixed vertical plate:

Consider a jet of water coming out from the nozzle strike a flat vertical plate —

- Let $V =$ Vel of the jet.
 $d =$ dia of the jet.
 $a =$ area of cross-section of the jet $= \frac{\pi}{4} d^2$.

As the plate is fixed, the jet after striking will get deflected through 90° .

Hence the component of the vel of jet, in the dirⁿ of jet after striking will be zero.

$$F_x = \text{Rate of change of momentum in the dirⁿ of force} - \text{Initial momentum} - \text{Final Momentum}$$

$$= \frac{\text{mass} \times \text{initial Vel} - \text{mass} \times \text{final Vel}}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{initial Vel} - \text{Final Vel}]$$

$$= \frac{\text{Mass}}{\text{Sec}} [\text{vel of jet before striking} - \text{vel of jet after striking}]$$

$$= \rho a v [v - 0]$$

$$= \rho a v^2$$

Note: - In the above eqn initial vel minus final vel is taken as bcz force exerted by the jet on the plate is calculated if force exerted on this jet is taken then final vel is taken.

Ex-1 Find the force exerted by a jet of water of dia 75 mm on a stationary flat plate when the jet strikes the plate normally with a vel of 20 m/s.

Soln → Data given as -

$$\text{Diameter of jet} = d = 75 \text{ mm} = 0.075 \text{ m}$$

$$\text{Velocity of jet} = v = 20 \text{ m/s}$$

$$\text{Area of the jet} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.075)^2 = 0.004414 \text{ m}^2$$

The force exerted by the jet of water on a stationary vertical plate is given by -

$$F = \rho a v^2 = 1000 \times 0.004414 \times (20)^2 = 1765.6 \text{ N}$$

Ex-2 Water is flowing through a pipe at the end of which a nozzle is fitted the dia of the nozzle is 100 mm and the head of the water the centre of nozzle is 100 m. Find the force exerted by the jet water on a fixed vertical plate. The Co. of vel is 0.95.

Soln → Data given as:

$$\text{Diameter of nozzle} = d = 100 \text{ mm} = 0.1 \text{ m}$$

$$\text{Head of water } H = 100 \text{ m}$$

$$\text{Co. of vel } C_v = 0.95$$

$$\text{Area of nozzle } (a) = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.1)^2 = 0.007854 \text{ m}^2$$

$$\text{Theoretical vel of jet of water is given as } V_{th} = \sqrt{2gH}$$

$$= \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

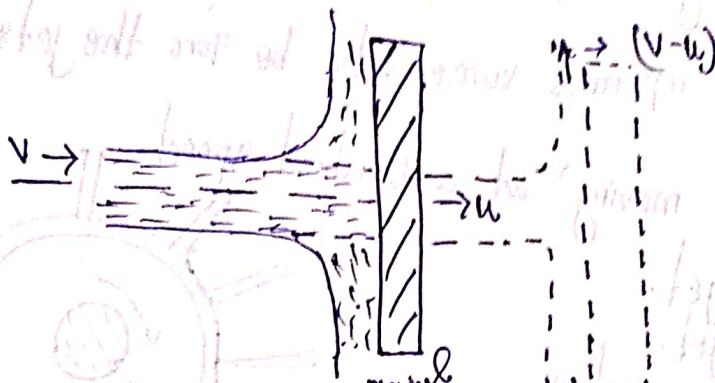
$$\text{But } C_v = \frac{\text{actual vel}}{\text{theoretical vel}}$$

Actual Vel of jet of water (v) = $C_v \times V_{th}$

Force exerted on a fixed vertical plate is given by —

$$F = \rho a v^2 = 1000 \times 0.07854 \times (12.08)^2 = 13907.2 \text{ N} = 13.9072 \text{ kN.}$$

Ans



Jet striking a flat vertical plate :

Consider a jet of water striking a flat plate moving with a uniform vel away from the jet —

- Let $V =$ Vel of the jet.
- $A =$ Area of cross-section of the jet.
- $u =$ Uniform vel of flat plate.

In this case the jet strikes the plate with a rel velocity which is equal to the abs vel of jet of water minus the vel of the plate.

Hence rel vel of the jet with respect to plate = $v - u$.

Mass of water striking the plate per sec —

$$= \rho \times \text{area of the jet} \times \text{Velocity} = \rho a \times [v - u]$$

Force exerted by the jet on the moving flat plate in the dirⁿ of motion of jet —

$$F_x = \text{Mass of water striking/sec} \times [\text{Initial Vel} - \text{Final Vel}]$$

$$= \rho a [v - u] [v - u - 0] = \rho a [v - u]^2$$

[Final vel in the dirⁿ of jet is zero.]

In this case the work will be done by the jet on plates as the plate is moving. Work done per sec by the jet on the plate —

$$= \text{Force} \times \frac{\text{distance in the dirⁿ of force}}{\text{time}}$$

$$= F_x \times u$$

$$= \rho a (v - u)^2 \times u$$

∴ 1 flat plate work done is zero

Jet strikes a Series of plates: — In this case a large no of flat plates are mounted on the rim of a wheel fixed distance apart.
 → The jet strikes a plate and due to the force exerted by the jet on the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on 2nd plate.

Thus each plate appears successively before the jets the jet exerts force on each plate.

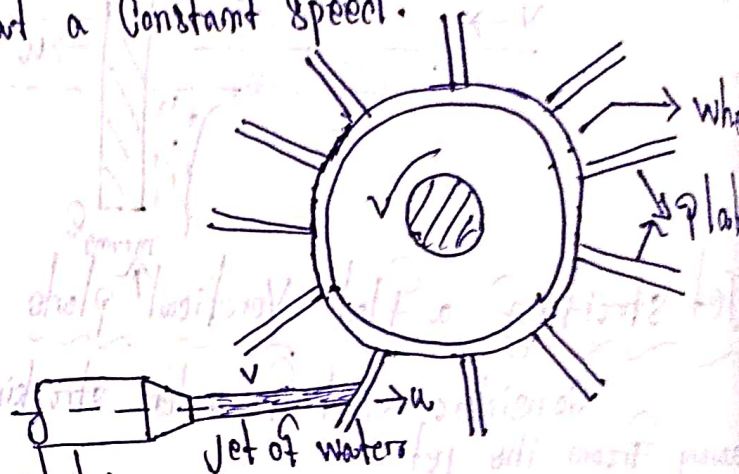
The wheel starts moving at a constant speed.

Let $V =$ Vel of the jet.

$D =$ dia of the jet.

$A =$ Area of cross-section.

$u =$ Vel of plate



In this case the mass of water coming out from nozzle per sec is always in contact with the plates when all the plates are considered.

→ Hence mass of water / sec = ρav .

The jet strikes the plate with $Vel = V - u$

The force exerted by the jet in the direction of the motion of plates

$$F_x = \frac{\text{mass}}{\text{Time}} [\text{Initial Vel} - \text{Final Vel}]$$

$$= \rho av [(V - u) - 0]$$

$$= \rho av [V - u]$$

Workdone per second by the jet on the series of the plates per sec

$$= F \times u = \rho av [V - u] u$$

Kinetic energy of the jet per second $\rightarrow \frac{1}{2} mv^2 = \frac{1}{2} (\rho av) v^2$

$$\text{Efficiency} = \frac{\text{Workdone/sec}}{\text{Kinetic energy/sec.}}$$

$$= \frac{\rho av (V - u) u}{\frac{1}{2} \rho av^3} = \frac{2u(V - u)}{v^2}$$

Condition For max efficiency — For a given jet velocity V , the efficiency will be max. when.

$$\frac{d}{dU} = 0, \Rightarrow \frac{d \left[\frac{2UCV - U^2}{V^2} \right]}{dU} = 0$$

$$\Rightarrow \frac{d \left[\frac{2UV - U^2}{V^2} \right]}{dU} = 0$$

$$\Rightarrow \frac{2V - 2U}{V^2} = 0$$

$$\Rightarrow 2V - 2U = 0 \Rightarrow \boxed{U = V/2}$$

Maximum Efficiency: —

Substituting the value of $V = 2U$.
we get max efficiency as

$$\text{max} = \frac{2U[2U - U]}{(2U)^2} = \frac{1}{2} = 50\%$$

Impact of jet on a moving curved plate when jet strikes tangentially at one of the tips: —

Consider a jet of water striking a moving curved vane tangentially at one of its tips.

In this case as plate is moving, the vel with which jet of plate is equal to the rel. vel of the jet w.r.t the plate.

Let V_1 = vel of the jet at inlet.

U_1 = vel " " plate " "

V_{r1} = Rel vel of the jet & plate at inlet.

α = guide blade angle.

θ = Vane angle made by rel vel V_{r1} with the dirⁿ of motion of inlet.

V_{w1} & V_{f1} = Components of V_1 in the dirⁿ of motion & perpendicular to the dirⁿ of motion of Vane respectively.

V_{w1} = whirl vel at inlet.

V_{f1} = vel at inlet.

V_2 = vel of jet at outlet.

U_2 = vel of plate at outlet.

V_{r2} = Rel vel of jet at outlet.

β = Angle made by vel V_2 with dirⁿ of motion of vane at outlet.

ϕ = Vane angle at outlet.

V_{w2} = vel of wheel at outlet.

V_{p2} = vel at outlet.

The triangle ABD & EAH are called the vel triangle at inlet & outlet. If the vane is smooth & having vel in the dirⁿ of motion at inlet & outlet equal we have —

$$V_1 = V_2 = U, \quad V_{r1} = V_{r2}$$

Now, Mass of water striking vane per sec

Where a = Area of the jet.

Forces exerted by the jet in the dirⁿ of motion —

$$F_x = \frac{\text{mass}}{\text{Time}} [\text{Initial Vel} - \text{Final Vel}]$$

But initial vel with which jet strikes the vane = V_{r1} .

The component of this vel in the dirⁿ of motion = $V_{r1} \cos \theta = (V_{w1} - U)$

Similarly the component of rel vel V_{r2} at outlet in the dirⁿ of motion

$$= -V_{r2} \cos \phi = -[U_2 + V_{w2}] \quad \text{[-ve sign is taken as Component } V_{r2} \text{ in opposite dirⁿ]}$$

Substituting these values in the above eqn —

$$F_x = \rho a V_{r1} [C V_{w1} - U] - [-C U_2 + V_{w2}]$$

$$= \rho a V_{r1} [V_{w1} - U + U_2 - V_{w2}]$$

$$= a V_{r1} [V_{w1} + V_{w2}]$$

This eqn is true only when B is acute when —

$$B = 90^\circ, \quad V_{w2} = 0, \quad F_x = a V_{r1} [V_{w1}]$$

$$\text{When } B > 90^\circ \text{ (obtuse)} \quad F_x = a V_{r1} [V_{w1} - V_{w2}]$$

$$\text{In eqn } F_x \text{ is written as } F_x = a V_{r1} [V_{w1} \pm V_{w2}]$$

Work done/sec on the vane by the jet —

$$F_x \times U = \rho a V_{r1} [V_{w1} \pm V_{w2}] \times U$$

Work done/sec unit weight of fluid striking/sec —

$$= \frac{\rho a V_{r1} [V_{w1} \pm V_{w2}] \times U}{\rho a V_{r1} \times g} = \frac{[V_{w1} \pm V_{w2}] \times U}{g}$$